

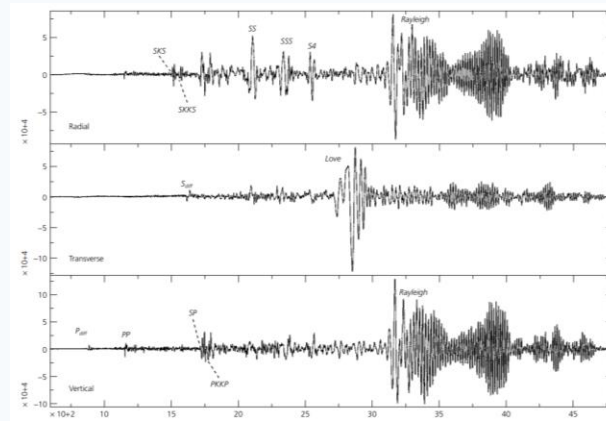
Surface wave tomography

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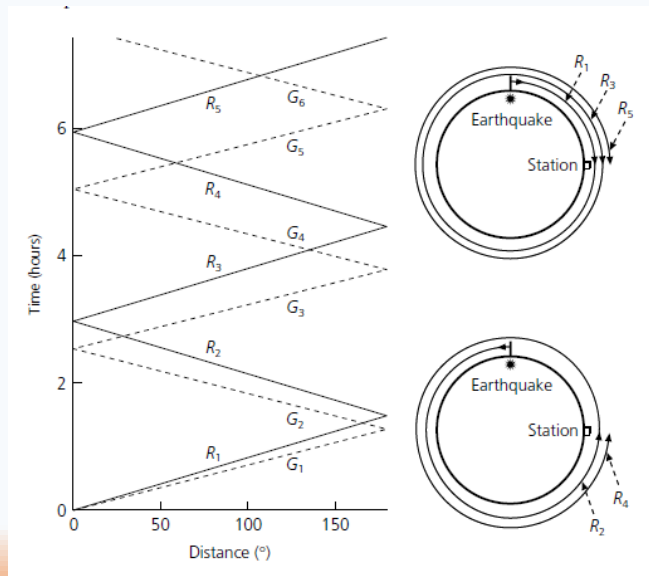
General flow of the presentation:

- What is a surface wave?
- Features of the surface waves and the different types
- Dispersion and higher modes
- Group and phase velocities
- An automated approach for the calculation of the dispersion curve
- Tomographic models and what impacts them
- Some issues

What is a surface wave?



A surface wave is a mechanical wave that forms at a surface with no parallel shear forces acting at the boundary. Surface waves often dominate the seismic waveforms after earthquakes shown here in the seismic waveform. This seismogram is a three component seismometer that has been rotated into the radial, transverse, and vertical components.

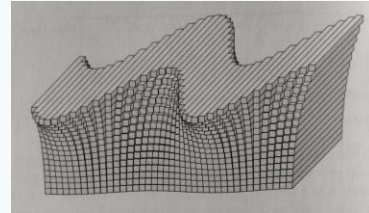
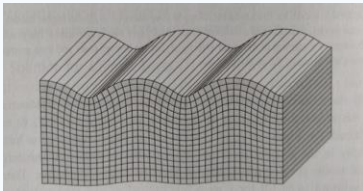


Due to the fact that the surface waves travel along the surface, rather than through a volume like body waves, they decay at a lower rate relative to r (distance from source) compared to the decay of body waves which decay at r^2 .

This leads to surface waves being detected multiple times from a single station following an earthquake. With major earthquakes ringing out for hours following a major earthquake.

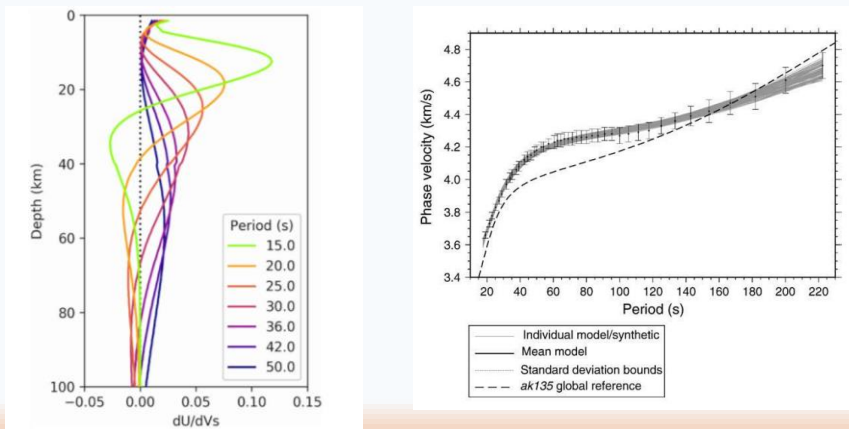
Types of Surface Wave

- Rayleigh and Love wave



The two types of surface waves are the Rayleigh wave and the Love wave. In the most basic model, Rayleigh waves are formed due to the constructive interference of P-waves and Sv-Waves (vertical shear) in an infinite half space and a free surface. While Love waves are created through the constructive interference of Sh-waves (Horizontal shear) in a layer with a free surface above and a infinite half space of a different density below. This layer structure causes a fundamental feature of surface waves, **dispersion**. Where different frequencies have different velocities due to the sensitivity to depth.

Dispersion



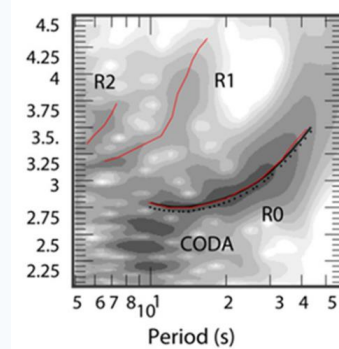
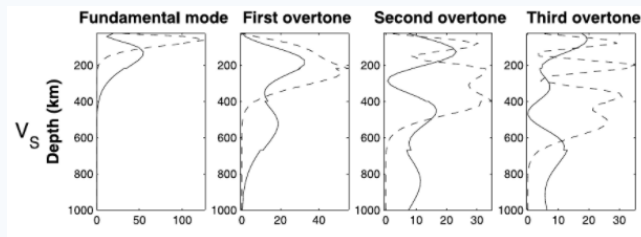
Gosseilin, J.M., et al, 2020
Darbyshire, F.B., 2013

Dispersion is when a change in the frequencies of the surface wave has a change in velocity. The sensitivity kernels on the left diagram demonstrate that these changes in velocities are due to different frequencies being sensitive to different depth within the subsurface. In general, longer periods (lower freq.) waves are more sensitive to deeper structures within the subsurface.

This allows us to calculate a 1D **absolute** velocity profile between an event and a station, (or two stations) based on one surface wavetrain, compared to a relative velocity along a ray that we would get through the use of body waves. This is seen on the right diagram in the form of a dispersion curve which is the change in surface wave velocities relative to depth, with hundreds/thousands of these curves being created for surface tomography studies.

However, as surface waves travel through the Earth, the sensitivity to structures decays as the change in velocities with respect to shear waves diminishes, shown by the sensitivity kernels. This leads to an effective depth of the fundamental mode surface waves to around 300 km depth. Which is when we start to look towards higher modes of waves.

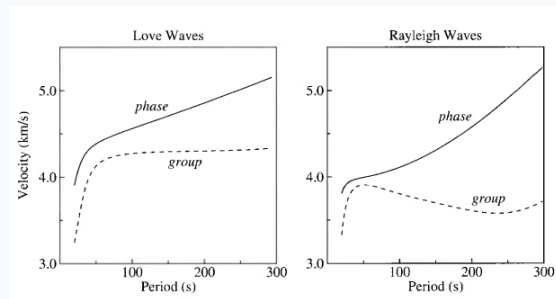
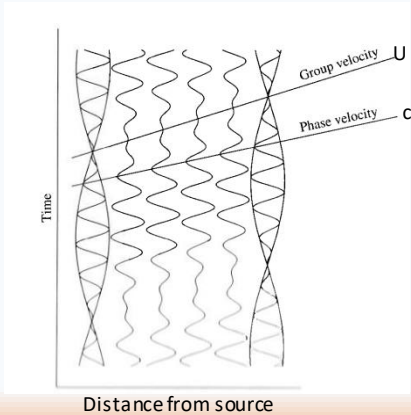
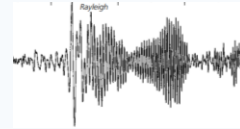
Modes



Ferreira, A.M.G., et al., 2010
Levshin A.L., et al., 2019

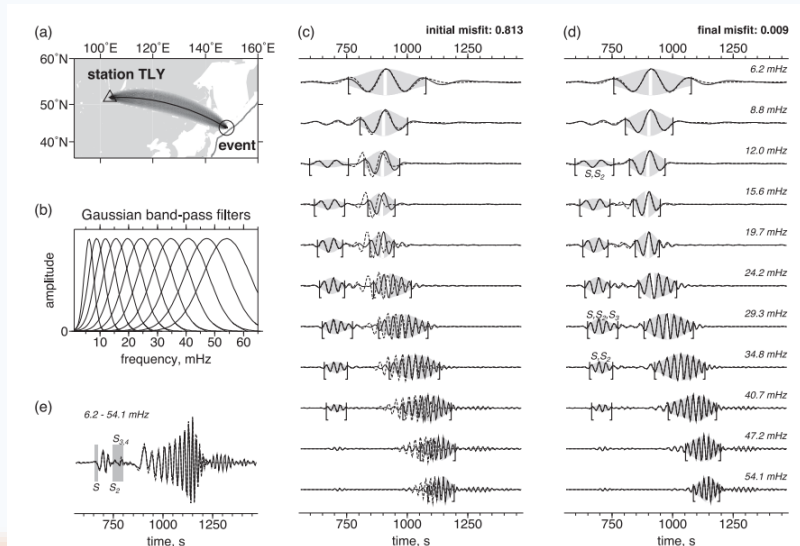
Higher modes (or overtones, or harmonics) are the higher frequency resonant frequencies of the fundamental mode, typically half the period of the fundamental wave (double the frequency) and allow us to sample deeper into the Earth. The left diagram: solid line in period:100s, and the dashed line is a period of 40s. You may see studies that talk about using higher modes and this is what they are referring too. However, these signals are often weaker the fundamental mode signal so can get convolved in the time domain. So we can view these higher modal waves more easily in the frequency domain.

Group vs phase velocities



For Surface waves, there are two potential types of velocities that could be measured for the estimation of the velocity structure of the mantle. The first is the group velocity and is the velocity of the envelope of energy from the seismic waveform. For this an idea of the timing of the earthquake and a general location is what is required, with the dispersion being calculated through a band pass filtering of the surface wave.

The other method is the phase velocity, which is the calculated velocity of the individual phases within the seismic wavetrain. This can be done by converting the seismic wave train to the frequency domain, or through an f - k analysis to calculate phase velocity. This is more computationally intensive but allows for better quality results than from group velocities. It also requires that the earthquake focal mechanism or moment tensor is known to establish the initial phase of the wave.



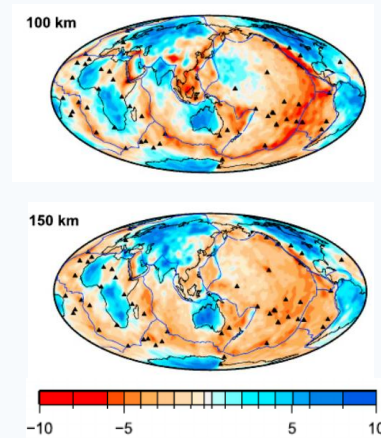
Schaeffer A.J., & Lebedev, S., 2013

This is an automated waveform inversion created by Schaeffer and Lebedev. For the inversion, the waveform data is initially compared to a synthetic waveform based on a background model for a region. This is done, at least in this technique, through the band pass filter of the waveform for multiple frequency ranges both in the synthetic and for the waveform data. A cross-correlation technique for the respective time windows calculates the misfit between the synthetic and the observed waveform data simultaneously across all the freq. bands. This is then inverted using a non-linear inversion technique, with this one making use of a calculated sensitivity function surrounding the ray path from source to receiver. This technique is used to minimise the misfit between the synthetic and the observed data points, thus creating a 1D dispersion curve between source and receiver.

Another technique could use a Monte-Carlo Markov Chain technique to calculate multiple dispersion curve velocity structures and then use the mean of the best fitting models, which is what is used on the dispersion curve on slide 5.

Tomographic models

- Parameterization
- Regularization
- Ray coverage
- Resolution



Debayle et al., 2016

Tomographic models are a 2D or 3D velocity model of the upper mantle, created using the cross cutting relationships of waves in the subsurface. For surface waves, these rays travel along a horizontal path, making them very good for vertical resolutions of the upper mantle. The nature of the dispersion also makes them very useful for regional or global studies, particularly in areas where placing a seismometer may be difficult, such as beneath oceans or beneath Antarctica.

However, surface waves do struggle with lateral density variation (especially compared to body waves) and without significant ray path coverage, lateral smearing can occur.

Another issue is that the non- uniqueness of the solution makes the tomographic inversion a complicated problem. Some methods can be used to make the problem more manageable and minimize the number of results. These are mainly attributed to the parameterization of the model as well as the regularisation. Other contributions that effect the tomographic image is the ray path coverage, and the addition of other datasets for joint inversion. All of these features will impact the relative resolution of features that the tomographic images can recover.

Parameterization

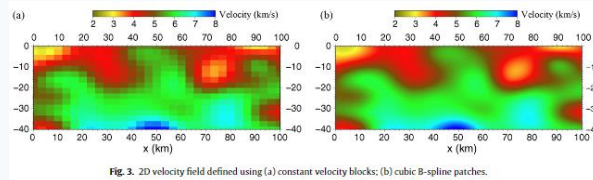
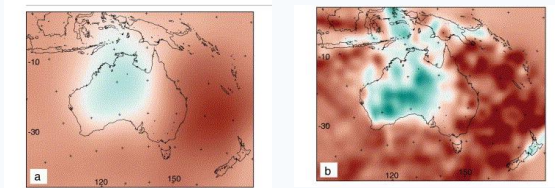


Fig. 3. 2D velocity field defined using (a) constant velocity blocks; (b) cubic B-spline patches.

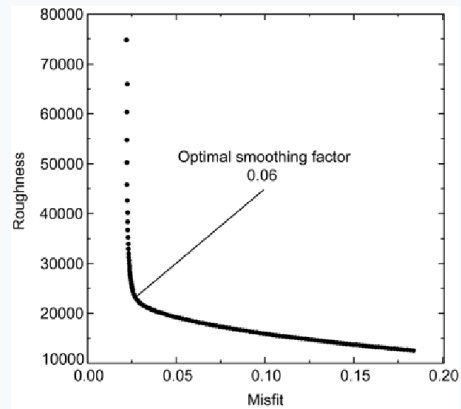
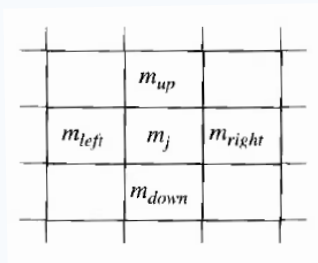


Rawlinson, N., et al., 2009
Fishwick, S., et al., 2005

Parameterization can effect how the problem is solved and the final look of the image. Models could use (though highly unlikely nowadays) a constant velocity block with all the elastic properties defined in a square or cube within the subsurface. This is the classic look of tomographical models in text books, but leads to poor estimations of velocities in the subsurface, assuming entire regions as one velocity, as well as forming discontinuities which tomographic models are not good at picking out, unless combined with another study, leading to a smeared velocity profile. More likely, the elastic and density parameters will be assigned to model points (or knots) and an interpolation technique used to calculate the resulting velocity structure, as well as the paths of seismic rays through the subsurface. These interpolation techniques could be splines, polynomials or surface waves. Another thing that may impact the image is the distance between modal points which will impact the effective resolution of recoverable structures in the subsurface. Bottom left: 8 degree knots, Right: 2 degree knots

Regularisation

- Least squares
- Laplacian average

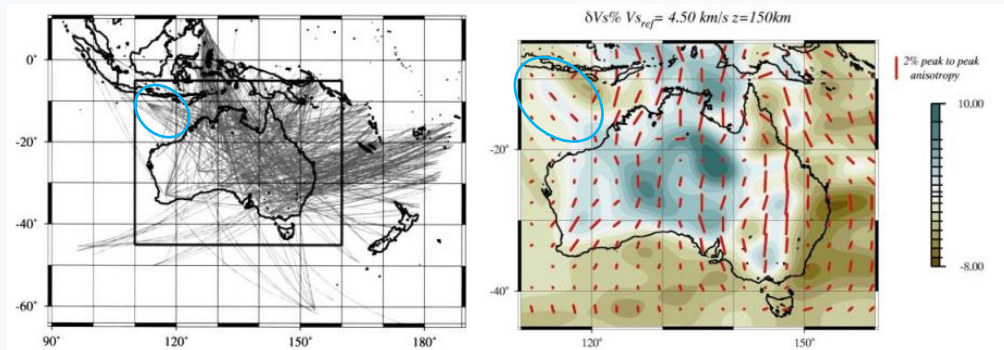


Shearer, P.M., 2019

Least squares approximation for working with data points with no ray path coverage
Laplacian method to deal with high variations between adjacent points. May impact

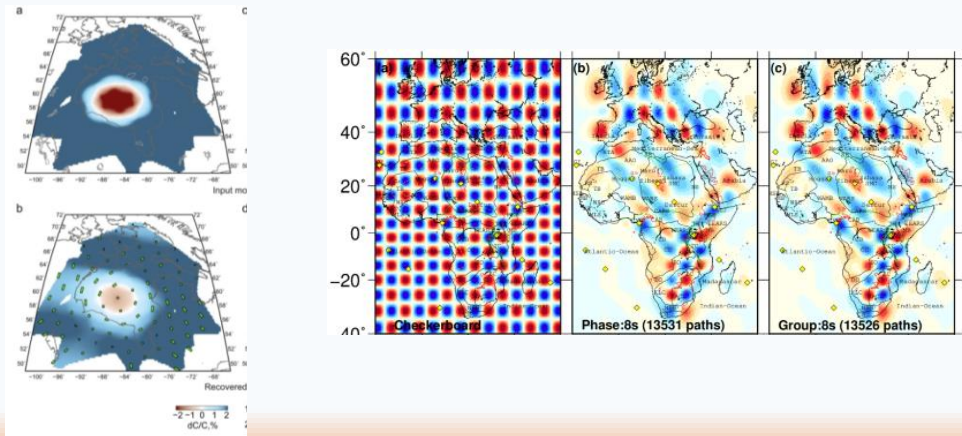
C value between the model roughness and the misfit

Ray coverage



While there vertical velocities can be accurately constrained, the lateral resolution is often poorer compared to body wave tomography
Due to the high variability in the crust, surface waves have trouble resolving structures closer to the surface.

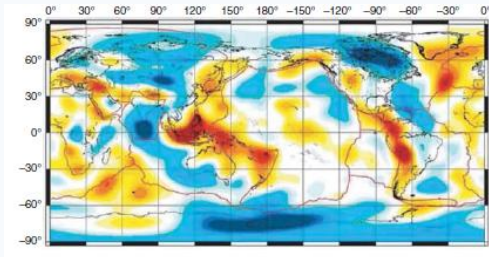
Resolution of the tomographic models



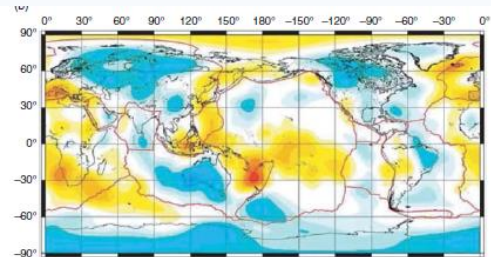
Ojo, A.O, et al., 2020
Darbyshire, F.B., 2013

Resolution models are a good way of presenting how well your models are at recovering structures in the subsurface. Either a feature map or a checkerboard test. This will show regions of damped anomalies where the true amplitude of the anomalies can not be effectively recovered, or regions of smearing in models (for surface waves this will most likely be a lateral smearing)

Some problems



EGM96 free-air gravity model



Seismic model predicted free-air gravity model

Afonso et al., 2016

This is an example where the seismic anomalies may not truly reflect the real structures, due to damped anomalies.

Thank you for listening

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